of this book is that the statements made do not require modification by implicit restrictions to be inferred from the context. The more fundamental the notions used, the more satisfactory is the clearcut language. We need more texts like this in America, books that can be used in every college, but which represent from elementary aspects the finest of mathematical scholarship.

A few minor infelicities may be mentioned: A complex number is discussed under two distinct representations. The less common is called explicitly (page 3) "the trigonometric form," while the other remains unnamed. Immediately after emphasizing the fact that a complex number has many amplitudes differing by integral multiples of 360°, the term, "the amplitude" is used without a hint as to which one is "the amplitude." The terms, "linear factor" and "factored form" are defined in the quadratic case, while the terms are then employed in connection with the general polynomial without further discussion. A product constituting a *polynomial* is called the factored form of an *equation.* In connection with mathematical induction the author speaks of "changing n into $n + 1$." The phrase, "upper limit to the real roots" is defined as a single concept without any inquiry as to the meaning of the word, "limit," or the sense of "upper," but in the exercises, the words "lower limit" are mentioned casually as though then familiar to the reader. "The numbers (10) are known as *Cardan's Formulas.*" The discriminant, after being defined in general on page 47, is redefined for the quartic on page 51. "A point on the graph at which the tangent is both horizontal and an ordinary tangent is a bend point. . . ." In the exercises on page 74 the letters must denote only real quantities although no such warning is given. But minor blemishes such as these and rare misprints do not seriously impair the usefulness of a simple but scholarly text.

ALBERT A. BENNETT.

Mathematical Philosophy. A study of fate and freedom. Lectures for educated laymen. By CASSIUS J. KEYSER. New York, E. P. Dutton & Company, 1922. 8vo. 14 + 466 pages. Price $4.70.

In the preface of this book its author expresses the hope that these lectures may not be ungrateful to the following two classes of readers, among others:

"To the growing class of such professional mathematicians as are not without interest in the philosophical aspects of their science. To the growing class of such teachers of mathematics as endeavor to make the spirit of their subject dominate its technique."

The lectures were designed primarily for students whose major interest is in philosophy, but the present review is restricted to a consideration of the merits of the book for the two classes of readers just noted. Such readers will find in this volume much that is inspiring, much that they will enjoy to re-read, much that will be instructive and will lead them to look at subjects from a new point of view. Comparatively few of these readers will probably find here the enduring qualities of real mathematics, but they will find certain views which will extend their horizon as regards the nature and bearing of real mathematics and which will enable them to present their subject in a more popular form.

The mathematical knowledge presupposed on the part of the reader is limited to those facts about algebra, geometry and trigonometry which a capable student can acquire in one collegiate year, but the more mature student of mathematics will evidently read with much more insight the chapters on such subjects as transformation, invariance, the group concept, variables and limits, infinity, hyperspace, and non-euclidean geometries. In fact, certain parts of the chapters just noted cannot be read with much satisfaction by mathematical students whose maturity is below that of the ordinary beginning graduate, but the less mature student is likely to be attracted by the style to read many things that he does not fully comprehend, and such reading is sometimes very profitable.

The first mathematical subject which receives serious attention is the postulates of geometry. The second lecture is explicitly devoted to this subject and in several of those which follow the same subject is more fully elucidated in a masterly manner. This does not imply that the critical mathematical readers will agree with the author as regards all the details. In particular, some of these readers will doubtless disagree with the following definitions found on page 114. "What I wish now to say is that any geometry built upon a postulate system containing Euclid's parallel-postulate, or its equivalent, is called Euclidean, however widely it may differ in other respects from Euclid's Elements; and, correspondingly, any geometry, like that of Lobachevski or that of Riemann, whose postulate system contains a contradictory of Euclid's parallel-postulate, is said to be non-Euclidean, no matter how much it may be like Euclid's Elements in other respects. Such are the specific and more usual senses in which these familiar adjectives are employed in the literature of geometry."

Many mathematicians would doubtless have substituted for this definition of euclidean geometry one based on the group concept as is done, for instance, on page 344 of tome 3, volume 1, of the *Encyclopédie des Sciences Mathématiques.* It is true that in the early period of the development of non-euclidean geometry only one geometry besides the euclidean seemed possible, as is also stated on page 3 of this encyclopedia, and the definitions quoted above have been extensively used. In fact, Professor Keyser would probably not wish that all his readers should agree with him in every case. His object seems to have been a higher one, as he dared to speak of many things in regard to which differences of opinion can normally be expected and in regard to which he did not claim expert knowledge. Some of his observations even on these subjects are valuable as the present writer can testify after re-reading the chapter of 33 pages on the group concept.

The teachers of mathematics who are mainly interested in enlarging their technical knowledge will doubtless find other books more helpful than the one under review, but those who are looking for a lucid exposition of some of the fundamental notions of mathematics, especially as regards their presence in other fields of human interest, will find here a richness which they cannot well afford to miss. Professor Keyser has for a long time stood at the head of American mathematicians as regards a certain type of popularization of mathematics, and

the present volume seems to embody his most significant efforts in this direction thus far. We hope others may follow for our subject needs popularization as may be seen from some of the attacks thereon.

G. A. MILLER.

Stereometrie. By KARL ROHN, with an introductory note by FELIX KLEIN. Leipzig, Robert Noske, 1922. xvi + 188 pages. Price in Germany, \$1, American currency.

This work was substantially ready for publication at the time of Professor Rohn's death, in August, 1920,[1] the necessary completion of the manuscript in minor details having been done by his friend and former pupil Dr. Friedrich Wünschmann. Dr. Rohn was himself a pupil of Professor Klein, and the latter, in his appreciative introduction, speaks highly of his skill in the field of geometry.

The work sets forth in succinct form the essential features of modern pro- ective geometry with respect to solids, thus extending the ordinary treatment of the projective properties of figures in a plane to those of three dimensions. It begins with a review of plane geometry (50 pages) and then considers the sphere, cylinder, and cone, proceeding later to the properties of conic sections and other plane figures in space.

The work shows a return to the better type of German bookmaking of pre-war days and will be welcomed by students of modern geometry as an aid to their advanced work in this field.

DAVID EUGENE SMITH.

NOTES.

Professor SOLOMON LEFSCHETZ, of the University of Kansas, is now one of the collaborators on the *Bulletin des Sciences Mathématiques*, Paris. In the issue for December, 1922, pages 417–424, the three reviews of recent publications are by him. They are of L. SILBERSTEIN, *The Theory of General Relativity and Gravitation*, University of Toronto Press, 1922; G. C. EVANS, *Functionals and their Applications*, American Mathematical Society, 1918; and O. VEBLEN, *Analysis Situs*, American Mathematical Society, 1922.

The *Bulletin des Sciences Mathématiques* for December, 1920 (series 2, volume 44) devotes forty-one pages (297–337) to a review by E. CARTAN of Sir THOMAS MUIR, *The Theory of Determinants in the Historical Order of Development*, volumes 1–3 (London, 1906, 1911, 1920, see this MONTHLY, *1920*, 419). In this review an account is given chapter by chapter of the contents of the three volumes, bringing the history of determinants down to 1880. The reviewer points out how difficult it is in mathematics and in all branches of science to determine the paternity of any important theorem or discovery, and the inestimable value of the work of Sir Thomas in securing for us this information in the case of determinants.

[1] See this MONTHLY, *1921*, 43.